IDS 702: Module 7.3

AR and MA models

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AR MODELS

- The most common time series model is called the autoregressive (AR) model.
- When only one lag matters, the zero-mean AR(1) model is

$$y_t = \phi y_{t-1} + \epsilon_t; \;\; \epsilon_t \sim N(0, \sigma^2).$$

- With a non-zero mean, we have

$$y_t = \mu + \phi y_{t-1} + \epsilon_t; \;\; \epsilon_t \sim N(0, \sigma^2).$$

- When the mean is non-zero, we can choose to de-mean (mean-center) the series and model that instead.
- In both cases, for the AR(1) we basically have a linear regression where the value of the outcome at time t depends on value of outcome at time $t-1$.
- ϕ is the autocorrelation.

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AR MODELS

- For the zero-mean AR(1) model,
  - $|\phi| < 1$ represents stationary time series.
  - $\phi = 1$ is a random walk.
  - $|\phi| > 1$ implies non-stationary, "explosive" models.
- A stationary AR(1) series varies around its mean, randomly wandering off away from the mean in response to the "input" values of the random ϵ_t series, but always returning to near the mean, and never "exploding" away for more than a short time.
- AR(1) series with $0 < \phi < 1$ represent short-term, positive correlations that would damp out exponentially if ϵ_t were zero.
- Negative values of ϕ represent short-term, negative correlations.

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AR MODELS

- Let's explore what AR(1) models look like via simulations.
- Move to the R script here.
- Note that
  - autocorrelations decay steadily with lags.
  - partial autocorrelations go to zero after lag p.

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AR MODELS

- For a zero mean AR(p) model, we have

$$y_t = \sum_{k=1}^{p} \phi_k y_{t-k} + \epsilon_t; \;\; \epsilon_t \sim N(0, \sigma^2).$$

- So that for a non-zero mean AR(p) model, we have

$$y_t = \mu + \sum_{k=1}^{p} \phi_k y_{t-k} + \epsilon_t; \;\; \epsilon_t \sim N(0, \sigma^2).$$

- AR(p) models are capable of adequately representing a wide range of observed behaviors in time series for large enough p.

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AR models: how many lags?

- Several ways to decide how many lags to include.
- Use graphical techniques
  - Look at partial autocorrelation plots.
  - Set p at lag where correlations become small enough not to be important.
- Use a model selection criterion like BIC.
- See section 8.6 of the assigned readings.
- Sometimes in time series data, the partial autocorrelations are small even at lag 1.
- In this case, it can be reasonable to skip autoregressive models and just use usual linear regression modeling approaches.

What if the series is not stationary?

- Sometimes transformations can make stationarity a reasonable assumption.
- Differencing (subtract lagged values from outcome at time t) also often help; changes over time are more likely to be stationary than the raw values.
- Including predictors can also help as we will see later with the melanoma example.
- There are other models for non stationary time series.

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AR(p): including predictors

- We also might want to account for serial correlation in regression modeling.
- Linear regression assumes independent errors across individuals.
- As we have already seen with the melanoma example, this may not be reasonable with time series data.
- With a single predictor x_t, we have

$$y_t = \mu + \sum_{k=1}^{p} \phi_k y_{t-k} + x_t + \epsilon_t; \;\; \epsilon_t \sim N(0, \sigma^2).$$

- That is, the value of outcome at time t depends on value of outcome at time $t-1, t-2, \ldots, t-k$, but also on the predictor x at time t.
- Easy to extend the model to multiple predictors.

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Model assumptions: stationarity

- Coefficients and regression variance do not change with time.
  - Apart from changes in explanatory variables, the behavior of the time series is the same at different segments of time.
  - Generally, no predictable patterns in the long term
- Diagnostics: check if patterns in residuals are similar across time.
- Tests:
  - Ljung-Box
  - Augmented Dickey–Fuller (ADF)
  - Kwiatkowski-Phillips-Schmidt-Shin (KPSS)
- Remedies:
  - Sometimes transformations (e.g., using logs) can make stationarity more reasonable.
  - Use time series models that allow for drifts.

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Model assumptions: others

- Other assumptions
  1. Linearity
  2. Independence of errors
  3. Equal variance
  4. Normality
- Diagnose using the same methods we used for linear regression.
- Remedies include transformations and model changes as we had before.

MA MODELS

- The zero-mean MA(1) model is

$$y_t = \phi\epsilon_{t-1} + \epsilon_t; \;\; \epsilon_t \sim N(0, \sigma^2).$$

- With a non-zero mean, we have

$$y_t = \mu + \phi\epsilon_{t-1} + \epsilon_t; \;\; \epsilon_t \sim N(0, \sigma^2).$$

- The value of the outcome at time t depends on the value of the deviation from the mean (the error term) at time $t-1$.
- For a zero mean MA(p) model, we have

$$y_t = \sum_{k=1}^{p} \phi_k \epsilon_{t-k} + \epsilon_t; \;\; \epsilon_t \sim N(0, \sigma^2).$$

- So that for a non-zero mean MA(p) model, we have

$$y_t = \mu + \sum_{k=1}^{p} \phi_k \epsilon_{t-k} + \epsilon_t; \;\; \epsilon_t \sim N(0, \sigma^2).$$

MA MODELS

- Let's explore what MA(1) models looks like via simulations. Move back to the same R script.
- Note that
  - Autocorrelations die off almost immediately after lag 1.
  - In MA(p) model, autocorrelations (mostly!) die off after lag p. May not be exact since autocorrelation measures correlation between the actual outcome at different time points.
  - Partial autocorrelations are not particularly useful.
- It is possible to write any stationary AR(p) model as an $\mathrm{MA}(\infty)$ model. The reverse result holds for some constraints on the MA parameters. See the reading material.

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Deciding models?

- Use autocorrelations and partial autocorrelations to help decide model.
- Steady decay on autocorrelations often implies AR.
- Non zero autocorrelations before lag p and zero after lag p often implies MA.
- Sometimes use both AR and MA error structure, called an ARMA model.
- Whenever we take differences in y values to ensure stationarity before fitting ARMA models, we have ARIMA models.

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What's next?

Move on to the readings for the next module!