

IDS 702: MODULE 6.3

UNCONFOUNDEDNESS AND OVERLAP

DR. OLANREWAJU MICHAEL AKANDE

OBSERVATIONAL STUDIES

- We will not focus on randomized experiments since most of the data you will have to analyze in practice are actually based on observational studies.
- In observational studies, we do not control or know the assignment mechanism.
- In addition, the presence of measured and unmeasured confounders can create unbalance between the groups.
- Again, to do causal inference, we have to make some structural (often untestable) assumptions, e.g. on the treatment assignment, for identifying causal effects.
- Once we have those general assumptions, we also usually have to make model assumptions to do the actual estimation.

ESTIMANDS

Once again, we will focus on the following estimands:

- The **average treatment effect (ATE)**:

$$\tau = \mathbb{E}[Y_i(1) - Y_i(0)].$$

- The **average treatment effect for the treated (ATT)**:

$$\tau = \mathbb{E}[Y_i(1) - Y_i(0) | W_i = 1].$$

- The **average treatment effect for the control (ATC)**:

$$\tau = \mathbb{E}[Y_i(1) - Y_i(0) | W_i = 0].$$

- For binary outcomes, **causal odds ratio (OR) or risk ratio (RR)**:

$$\tau = \frac{\Pr[Y_i(1) = 1] / \Pr[Y_i(1) = 0]}{\Pr[Y_i(0) = 1] / \Pr[Y_i(0) = 0]}.$$

ESTIMANDS

- The relationship between ATE, ATT and ATC is given by

$$ATE = \Pr[W_i = 1] \cdot ATT + \Pr[W_i = 0] \cdot ATC$$

- In randomized experiments, ATT is equivalent to ATC because treatment and control groups are similar/comparable.
- ATE is then also equivalent to ATT (and ATC).
- In observational studies, ATE is usually different from ATT and ATC.
- The above relation does not hold for ratio estimands.

ASSUMPTIONS: UNCONFOUNDEDNESS

We will need two major assumptions (in addition to SUTVA). The first, we already talked about, that is,

Assumption 1: **Unconfoundedness**

$$Y_i(0), Y_i(1) \perp W_i | X_i,$$

or using the equivalent form from last class,

$$\Pr[W_i = 1 | X_i, Y_i(0), Y_i(1)] = \Pr[W_i = 1 | X_i]$$

- Assumes that within subgroups defined by values of observed covariates, the treatment assignment is random.
- Rules out unobserved confounders.
- Randomized experiments satisfy unconfoundedness.
- Untestable in most observational studies, but sensitivity can be checked.

IMPLICATIONS OF UNCONFOUNDEDNESS

- Under unconfoundedness, it turns out that

$$\Pr[Y(w)|X] = \Pr[Y^{\text{obs}}|X, W = w] \quad w = 0, 1.$$

- That is, the observed distribution of Y in treatment arm $W = w$ equals the distribution of the potential outcomes $Y(w)$.

Why does this matter or how does this help us?

- Well, the causal estimands are essentially expectations and probabilities.
- Recall again that ATE is

$$\text{ATE} = \mathbb{E}[Y_i(1) - Y_i(0)].$$

- ATE can then be estimated from the observed data using

$$\text{ATE} = \mathbb{E}_X (\mathbb{E}[Y^{\text{obs}}|X, W = 1] - \mathbb{E}[Y^{\text{obs}}|X, W = 0]).$$

- Note that we need to average out over the distribution of X since the original formula for ATE does not depend on any X .

ASSUMPTIONS: OVERLAP

Assumption 2: **Overlap (or positivity)**

$$0 < \Pr[W_i = 1 | X_i] < 1, \text{ for all } i.$$

- Notice that this is the probabilistic assignment from last class, that is,

$$0 < \Pr[W_i = 1 | X_i, Y_i(0), Y_i(1)] < 1.$$

- However, we can exclude $\{Y_i(0), Y_i(1)\}$ now because of the unconfoundedness assumption.

- $$e(x) = \Pr[W_i = 1 | X_i = x]$$

is usually called the **propensity score**.

IMPLICATIONS OF OVERLAP

- Overlap implies that, in large samples, for all possible values of the covariates, there are both treated and control units.
- This is important within the potential outcomes (or counterfactual) framework both conceptually and operationally (variance inflation).
- Unlike unconfoundedness, overlap can be directly checked from the data often using the estimated propensity scores.
- Unconfoundedness and positivity jointly define the **strong ignorability** assumption.

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WHAT'S NEXT?

MOVE ON TO THE READINGS FOR THE NEXT MODULE!