

IDS 702: MODULE 6.1

THE POTENTIAL OUTCOMES FRAMEWORK AND CAUSAL ESTIMANDS

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CAUSALITY



We do not have knowledge of a thing until we have grasped its why, that is to say, **its cause**.



-- Aristotle, Physics

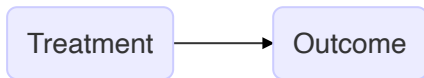
- Over the next few modules, we will discuss causal inference, specifically, on measuring the **effects of causes**.
- For now, we will simply lay the foundations for causal inference.
- We will get more into the actual methods later.

ASSOCIATION VS. CAUSATION

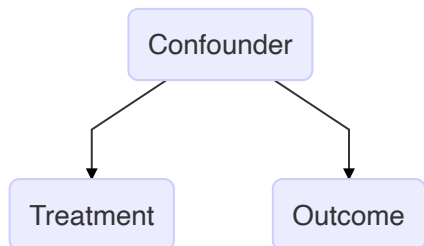
- In the models we have covered so far, our focus has been on inferring **associations** using samples drawn from our population of interest.
- For example, we have been asking questions such as, do people who receive job training tend to earn more wages than people who do not?
- Causal inference goes further as we try to infer aspects of the actual data generating process, that is, **causation**.
- For example, does receiving job training actually cause one to earn more wage than they would have without the training?
- The additional information needed to move from association to causation is often provided by **causal assumptions** (often untestable).
- Note: in most cases, **association does not imply causation!**

CONFOUNDING

- Why is it that association does not often imply causation? **confounding variables or confounders!**
- Causal relationship



- Confounding



EXAMPLES OF CONFOUNDING

- Ice cream consumption and number of people who drowned.
Confounder: temperature; people tend to consume more ice cream and also swim more when it is hot.
- Medical treatment and patient outcome.
Confounders: age, sex, other complications
- Education and income.
Confounder: socio-economic status of family
- An extreme example of confounding is Simpson's paradox: where a confounder reverses the sign of the correlation between treatment and outcome

SIMPSON'S PARADOX

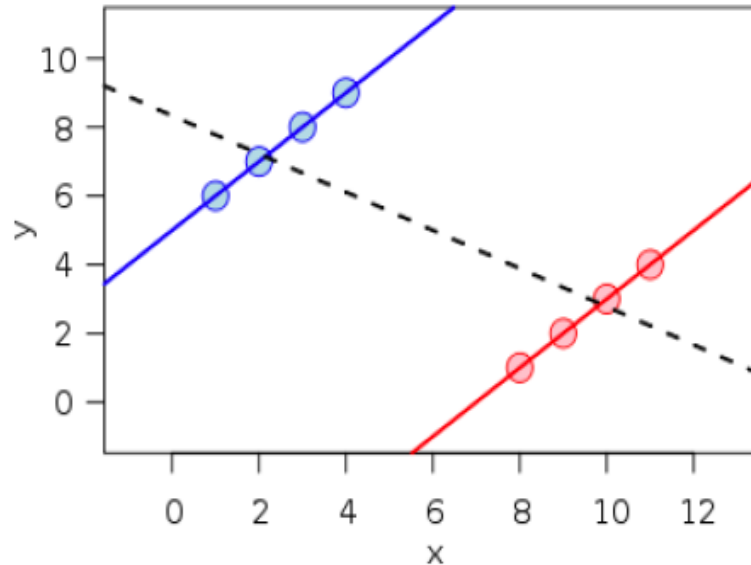
- Example: kidney stone treatment (Charig et al., BMJ, 1986).
 - Compare the success rates of two treatments for kidney stones
 - Treatment A: open surgery. Treatment B: small puncture

	Treatment A	Treatment B
Small stones	93% (81/87)	87% (234/270)
Large stones	73% (192/263)	69% (55/80)
Both	78% (273/350)	83% (289/350)

- Overall treatment B has a higher success rate but treatment A actually has higher success rates given the type of stones.
- What is the confounder here? Severity of the case/type of stones.

SIMPSON'S PARADOX OR YULE-SIMPSON EFFECT

- Simpson's paradox: a trend appears in different groups of data but disappears or reverses when these groups are combined.



- Mathematically, it is about conditioning.
- Another well-known example is the Berkeley admission gender bias (Bickel et al., Science, 1976).

GENERAL NOTATION

- **W**: Treatment (e.g. job training); we will focus on binary treatments.
- **Y**: Outcome (e.g. annual wages).
- **X**: Observed predictors or confounders (e.g. age, education, etc).
- **U**: Unobserved predictors or confounders.

- Examples of causal questions:
 - Causal effect of exposure to a disease.
 - Comparative effectiveness research such as in clinical trials: whether one drug or medical procedure is better than the other.
 - Program evaluation in economics and policy.

POTENTIAL OUTCOMES FRAMEWORK

POTENTIAL OUTCOMES FRAMEWORK

- The **potential outcomes framework** or **counterfactual framework** or **Rubin Causal Model (RCM)** is arguably the most widely used framework across many disciplines, e.g., medicine, health care, policy, social sciences.
- Under this framework, causal inference is viewed as a problem of missing data with explicit mathematical modeling of the assignment mechanism as a process for revealing the observed data.
- Rooted in the statistical work on randomized experiments by Fisher (1918, 1925) and Neyman (1923), as extended by Rubin (1974, 1976, 1977, 1978, 1990).

POTENTIAL OUTCOMES FRAMEWORK

- For a binary treatment, each individual gets exactly one of the two options, and we observe the corresponding response for that.
- Conceptually, under the potential outcomes framework, we think about what each individual's response should have been had they gotten the other treatment option instead of the one they actually got.
- The individual causal effect then is the difference between the two "potential" outcomes, only one of which is observed.
- Clearly, we never observe the two potential outcomes for any individual, making it natural to think of this as a missing data problem.

POTENTIAL OUTCOMES FRAMEWORK

- No causation without manipulation - "cause" must be (hypothetically speaking) something we can manipulate. e.g., intervention, action, treatment.
- That is, gender, time and age are not well defined "causes" under the RCM.
- Three integral components of the potential outcomes framework:
 - **potential outcomes** corresponding to the various levels of a treatment.
 - **assignment mechanisms**, that is, the treatment indicator for all observations.
 - a **model** for the science (the potential outcomes and covariates).

POTENTIAL OUTCOMES FRAMEWORK: BASIC CONCEPTS

- **Unit:** The person, place, or thing upon which a treatment will operate, at a particular time (note: a single person, place, or thing at two different times comprises two different units).
- **Treatment:** An intervention, the effects of which (on some particular measurement of the units) the investigator wishes to assess relative to no intervention (i.e., the control).
- **Potential Outcomes:** The values of a unit's measurement of interest after (a) application of the treatment and (b) non-application of the treatment (i.e., under control).
- **Causal Effect:** For each unit, the comparison of the potential outcome under treatment and the potential outcome under control.

CAUSAL EFFECTS

- For a single unit, let $Y(0)$ denote the outcome given the control treatment and $Y(1)$, the outcome given the active treatment.
- For example, suppose Y denotes a score (level of severity) for headache, then for a single unit, we could have

Raw scores				
Unit	Initial headache	Potential outcomes		Causal effect
	X	Y(asp)	Y(not)	Y(asp) - Y(not)
you	80	25	75	-50

Gain scores				
Unit	Initial headache	Potential outcomes		Causal effect
	X	Y(asp) - X	Y(not) - X	[Y(asp) - X] - [Y(not) - X]
you	80	-55	-5	-50

THE FUNDAMENTAL PROBLEM OF CAUSAL INFERENCE

As mentioned before,

- The fundamental problem of causal inference: we can observe at most one of the potential outcomes for each unit.
- Causal inference under the potential outcome framework is essentially a missing data problem.
- To identify causal effects from observed data, under any mathematical framework, one must make assumptions (structural or/and stochastic)
- Since we can at most observe a single potential outcome, we must rely on multiple units (and a lot of assumptions) to make causal inferences.

BASIC SETUP

- **Target population:** a well-defined population of individuals whose outcomes are going to be compared
- **Data:** a random sample of N units from a target population.
- A treatment with two levels: $w = 0, 1$.
- For each unit i , we observe
 - the binary treatment status $W_i \in \{0, 1\}$,
 - a vector of p predictors/covariates $X_i = (X_{i1}, \dots, X_{ip})$, and
 - an outcome Y_i^{obs} .

BASIC SETUP

- For each unit i , there are two potential outcomes $(Y_i(0), Y_i(1))$.
- That is, the outcomes under the two values of the treatment, at most one of which is observed.
- Potential outcomes and assignments jointly determine the values of the observed outcomes

$$Y_i^{\text{obs}} \equiv Y_i(W_i) = W_i \cdot Y_i(1) + (1 - W_i) \cdot Y_i(0)$$

and the missing outcomes:

$$Y_i^{\text{mis}} \equiv Y_i(1 - W_i) = (1 - W_i) \cdot Y_i(1) + W_i \cdot Y_i(0)$$

CAUSAL ESTIMANDS

- The **average treatment effect (ATE)**:

$$\tau = \mathbb{E}[Y_i(1) - Y_i(0)].$$

- The **average treatment effect for the treated (ATT)**:

$$\tau = \mathbb{E}[Y_i(1) - Y_i(0) | W_i = 1].$$

- The **average treatment effect for the control (ATC)**:

$$\tau = \mathbb{E}[Y_i(1) - Y_i(0) | W_i = 0].$$

- For binary outcomes, **causal odds ratio (OR) or risk ratio (RR)**:

$$\tau = \frac{\Pr[Y_i(1) = 1] / \Pr[Y_i(1) = 0]}{\Pr[Y_i(0) = 1] / \Pr[Y_i(0) = 0]}.$$

- Obviously these estimands are not identifiable without further assumptions.
- We will start to explore those soon.

EXAMPLE

	Potential Outcomes			Observed Data		
	Y(0)	Y(1)		W	Y(0)	Y(1)
	13	14		1	?	14
	6	0		0	6	?
	4	1		0	4	?
	5	2		0	5	?
	6	3		0	6	?
	6	1		0	6	?
	8	10		1	?	10
	8	9		1	?	9
True averages	7	5	Observed averages		5.4	11

ACKNOWLEDGEMENTS

These slides contain materials adapted from courses taught by Dr. Fan Li.

WHAT'S NEXT?

MOVE ON TO THE READINGS FOR THE NEXT MODULE!