IDS 702: Module 4.5

Multilevel/hierarchical logistic models

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Multilevel linear models

- The same idea and approach used to build multilevel models for normal data can be used to build multilevel logistic (and probit) models for binary outcomes.

- Recall that for a varying-intercepts linear model with one individual-level predictor, we have

$$\begin{aligned}
y_{ij} &= \beta_0 + \gamma_{0j} + \beta_1 x_{1ij} + \epsilon_{ij}; \quad i = 1, \ldots, n_j; \quad j = 1, \ldots, J; \\
\epsilon_{ij} &\sim N(0, \sigma^2); \\
\gamma_{0j} &\sim N(0, \tau_0^2)
\end{aligned}$$

  where x_{1ij} can be replaced with x_{1j} for a group-level predictor.

- This model is a compromise between complete pooling across groups of a grouping variable, such as counties in the radon example for last class (that is, same intercept for each county), and no pooling (estimating a separate intercept for each county without borrowing information).

- The degree of pooling is determined by the amount of information within and between groups.

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Multilevel logistic models

- We can use the same idea to build a varying-intercepts logistic model.
- That is,

$$
\begin{aligned}
y_{ij}|x_{ij} &\sim \text{Bernoulli}(\pi_{ij}); \quad i = 1, \ldots, n_j; \quad j = 1, \ldots, J; \\
\log\left(\frac{\pi_{ij}}{1-\pi_{ij}}\right) &= \beta_0 + \gamma_{0j} + \beta_1 x_{1ij}; \\
\gamma_{0j} &\sim N(0, \sigma_0^2)
\end{aligned}
$$

  where once again, x_{1ij} is an individual-level predictor which can be replaced with x_{1j} for a group-level predictor.

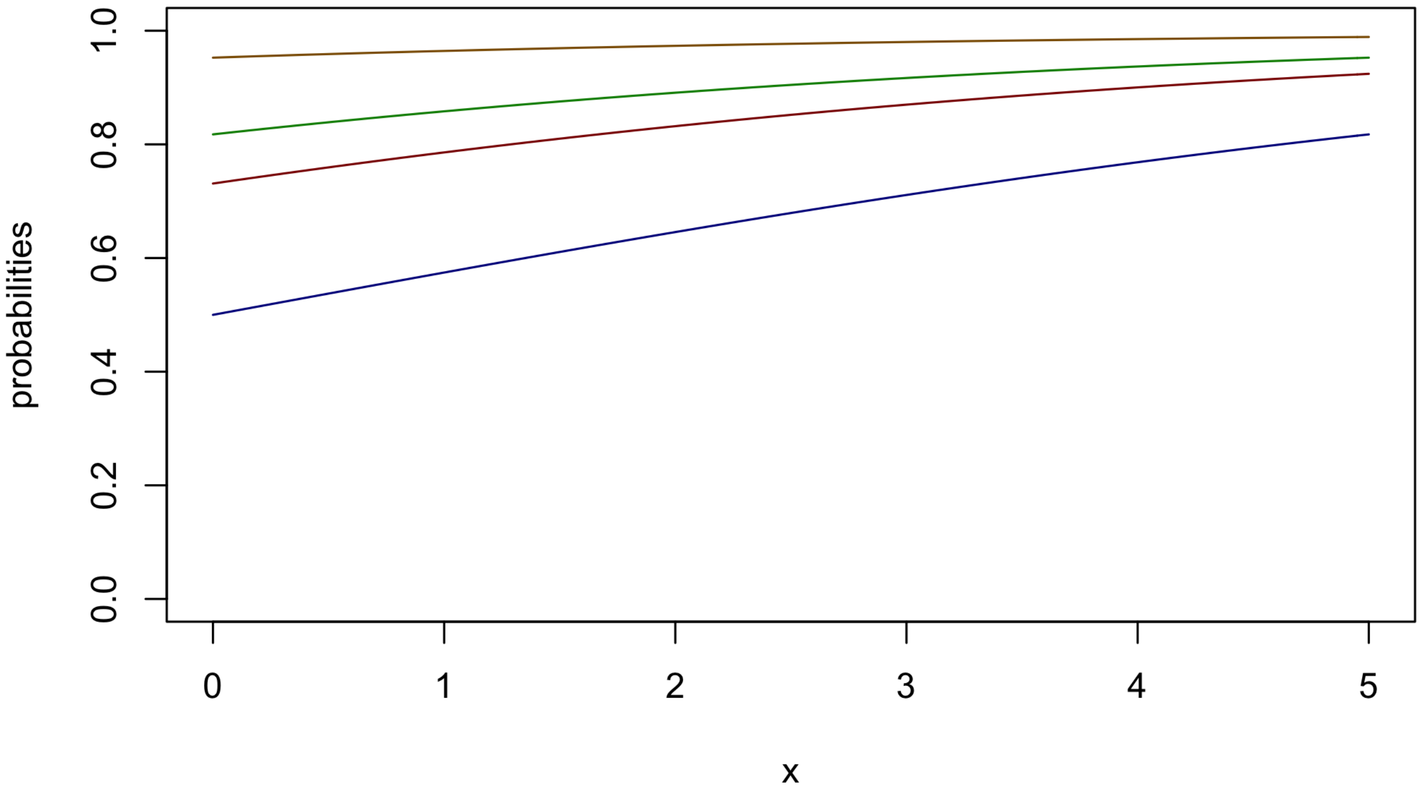
- The Gelman and Hill book uses the following notation instead

$$
\begin{aligned}
y_i|x_i &\sim \text{Bernoulli}(\pi_i); \quad i = 1, \ldots, n; \quad j = 1, \ldots, J; \\
\log\left(\frac{\pi_i}{1-\pi_i}\right) &= \beta_0 + \gamma_{0j[i]} + \beta_1 x_{i1}; \\
\gamma_{0j} &\sim N(0, \sigma_0^2).
\end{aligned}
$$

- I will use this notation in this module and the next.

Varying-intercepts logistic model

Inverse logit functions for varying-intercepts logistic models.



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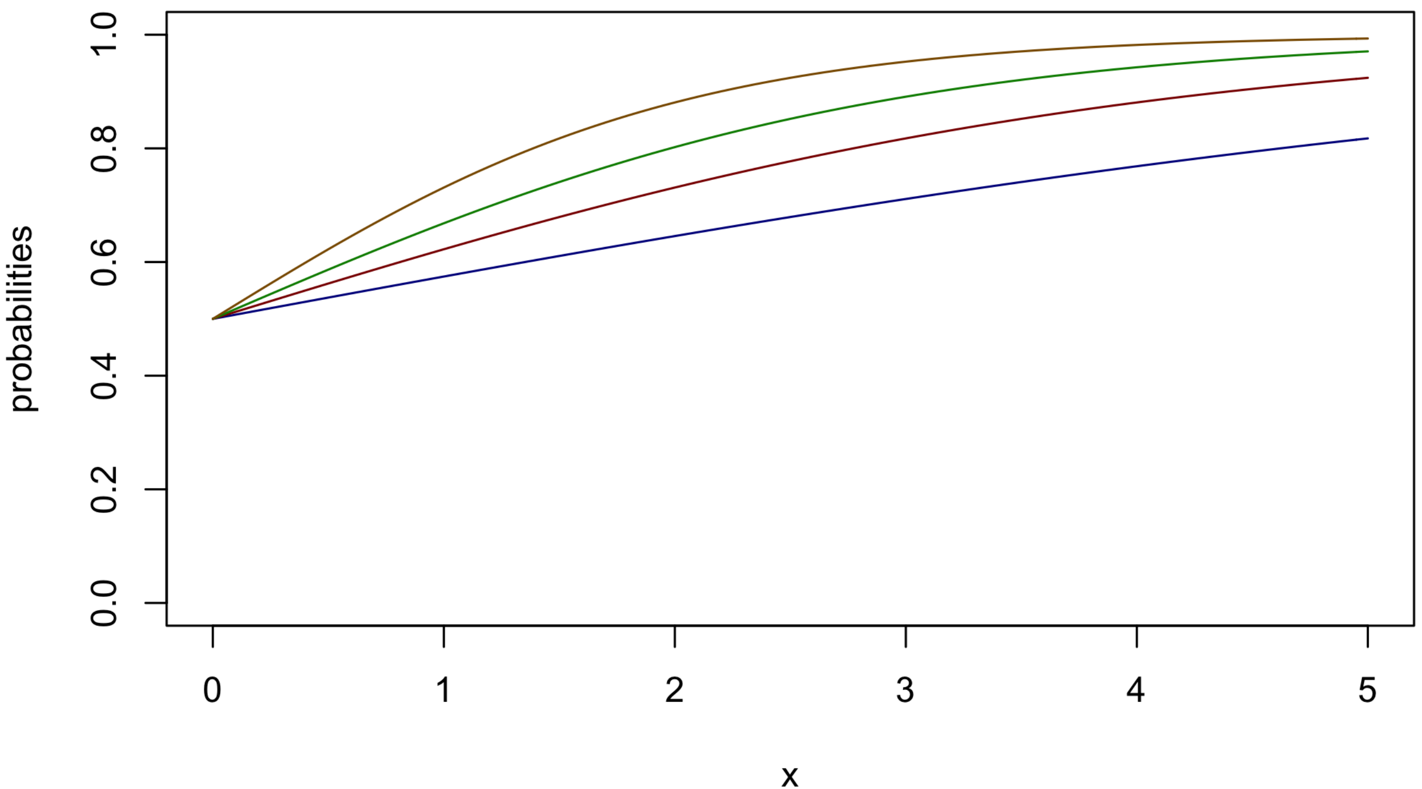
Multilevel logistic models

- It is easy to extend this model to allow for varying-slopes or both varying-intercepts and varying-slopes just like we had for multilevel linear models.
- The interpretations of the fixed effect(s) in multilevel logistic models follow directly from what we had for the standard logistic models, that is, log-odds, odds and odds ratios.
- The only difference now is the hierarchy in our data which allows us to borrow information across groups.
- One way to think about this is that we expect odds and odd-ratios to be more similar for observations within the same group, but we allow for some similarity across groups via partial pooling.

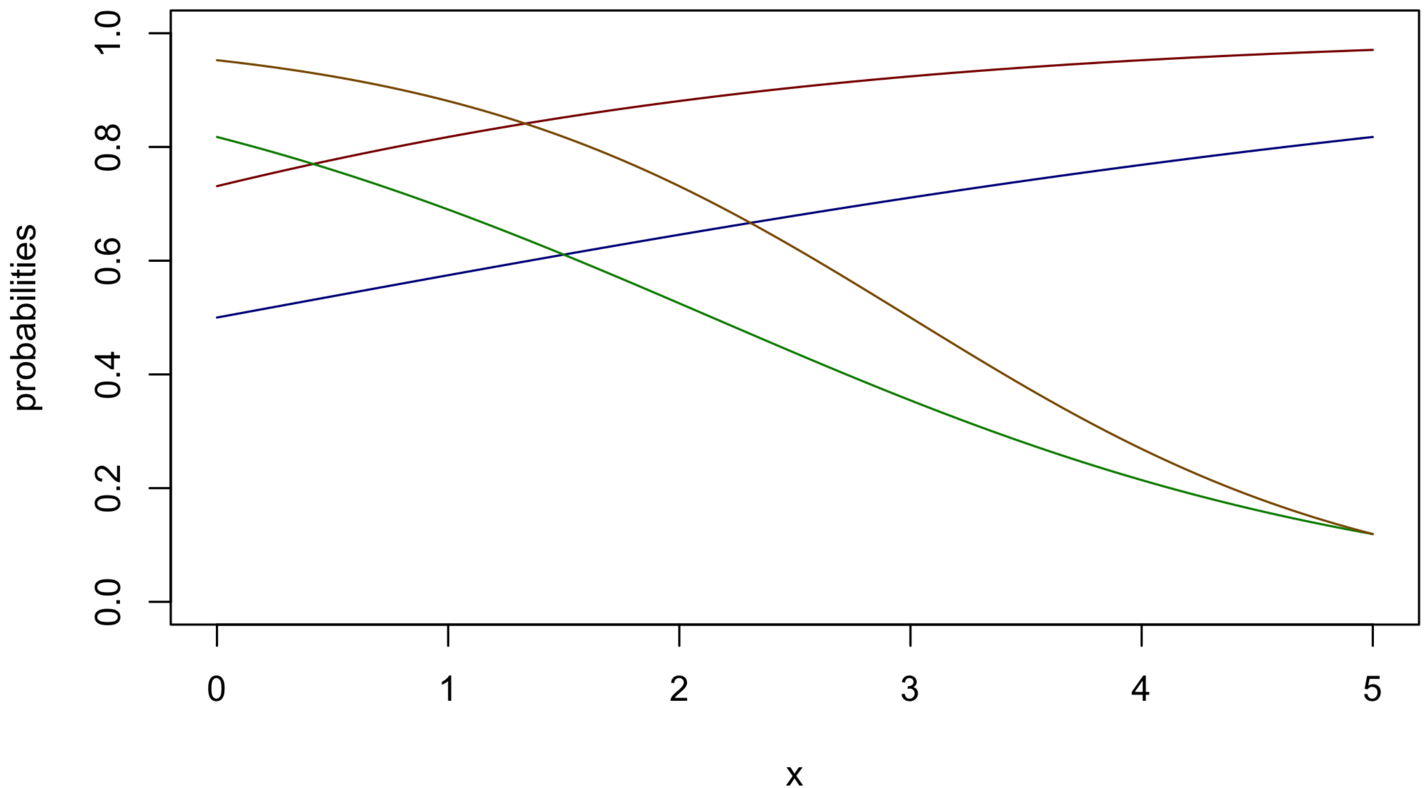
Varying-slopes logistic model

Inverse logit functions for varying-slopes logistic models.



Varying-intercepts, varying-slopes logistic model

Inverse logit functions for varying-intercepts, varying-slopes logistic models.



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1988 ELECTIONS ANALYSIS

- To illustrate how to fit and interpret the results of multilevel logistic models, we will use a sample data on election polls.
- National opinion polls are conducted by a variety of organizations (e.g., media, polling organizations, campaigns) leading up to elections.
- While many of the best opinion polls are conducted at a national level, it can also be often interesting to estimate voting opinions and preferences at the state or even local level.
- Well-designed polls are generally based on national random samples with corrections for nonresponse based on a variety of demographic factors (e.g., sex, ethnicity, race, age, education).
- The data is from CBS News surveys conducted during the week before the 1988 election.
- Respondents were asked about their preferences for either the Republican candidate (Bush Sr.) or the Democratic candidate (Dukakis).

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The dataset includes 2193 observations from one of eight surveys (the most recent CBS News survey right before the election) in the original full data.

Variable	Description
org	cbsnyt = CBS/NYT
bush	1 = preference for Bush Sr., 0 = otherwise
state	1-51: 50 states including DC (number 9)
edu	education: 1=No HS, 2=HS, 3=Some College, 4=College Grad
age	1=18-29, 2=30-44, 3=45-64, 4=65+
female	1=female, 0=male
black	1=black, 0=otherwise
region	1=NE, 2=S, 3=N, 4=W, 5=DC
v_prev	average Republican vote share in the three previous elections (adjusted for home-state and home-region effects in the previous elections)

Given that the data has a natural multilevel structure (through `state` and `region`), it makes sense to explore multilevel models for this data.

We will do just that in the next module.

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- Both voting turnout and preferences often depend on a complex combination of demographic factors.
- In our example dataset, we have demographic factors such as biological sex, race, age, education, which we may all want to look at by state, resulting in $2 \times 2 \times 4 \times 4 \times 51 = 3264$ potential categories of respondents.
- We may even want to control for `region`, adding to the number of categories.
- Clearly, without a very large survey (most political survey poll around 1000 people), we will need to make assumptions in order to even obtain estimates in each category.
- We usually cannot include all interactions; we should therefore select those to explore (through EDA and background knowledge).
- The data is in the file `polls_subset.txt` on Sakai.

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```r
###### Load the data
polls_subset <- read.table("data/polls_subset.txt",header=TRUE)
str(polls_subset)
```

```
## 'data.frame':    2193 obs. of  10 variables:
##  $ org   : Factor w/ 1 level "cbsnyt": 1 1 1 1 1 1 1 1 1 1 ...
##  $ survey: int  9158 9158 9158 9158 9158 9158 9158 9158 9158 9158 ...
##  $ bush  : int  NA 1 0 0 1 1 1 1 0 0 ...
##  $ state : int  7 39 31 7 33 33 39 20 33 40 ...
##  $ edu   : int  3 4 2 3 2 4 2 2 4 1 ...
##  $ age   : int  1 2 4 1 2 4 2 4 3 3 ...
##  $ female: int  1 1 1 1 1 1 0 1 0 0 ...
##  $ black : int  0 0 0 0 0 0 0 0 0 0 ...
##  $ region: int  1 1 1 1 1 1 1 1 1 1 ...
##  $ v_prev: num  0.567 0.527 0.564 0.567 0.524 ...
```

```r
head(polls_subset)
```

```
##      org survey bush state edu age female black region    v_prev
## 1 cbsnyt   9158   NA     7   3   1      1     0      1 0.5666333
## 2 cbsnyt   9158    1    39   4   2      1     0      1 0.5265667
## 3 cbsnyt   9158    0    31   2   4      1     0      1 0.5641667
## 4 cbsnyt   9158    0     7   3   1      1     0      1 0.5666333
## 5 cbsnyt   9158    1    33   2   2      1     0      1 0.5243666
## 6 cbsnyt   9158    1    33   4   4      1     0      1 0.5243666
```

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```
summary(polls_subset)
```

```
##      org           survey          bush            state            edu       
##  cbsnyt:2193   Min.   :9158   Min.   :0.0000   Min.   : 1.00   Min.   :1.000  
##                1st Qu.:9158   1st Qu.:0.0000   1st Qu.:14.00   1st Qu.:2.000  
##                Median :9158   Median :1.0000   Median :26.00   Median :2.000  
##                Mean   :9158   Mean   :0.5578   Mean   :26.11   Mean   :2.653  
##                3rd Qu.:9158   3rd Qu.:1.0000   3rd Qu.:39.00   3rd Qu.:4.000  
##                Max.   :9158   Max.   :1.0000   Max.   :51.00   Max.   :4.000  
##                               NA's   :178                                     
##       age            female           black             region     
##  Min.   :1.000   Min.   :0.0000   Min.   :0.00000   Min.   :1.000  
##  1st Qu.:2.000   1st Qu.:0.0000   1st Qu.:0.00000   1st Qu.:2.000  
##  Median :2.000   Median :1.0000   Median :0.00000   Median :2.000  
##  Mean   :2.289   Mean   :0.5887   Mean   :0.07615   Mean   :2.431  
##  3rd Qu.:3.000   3rd Qu.:1.0000   3rd Qu.:0.00000   3rd Qu.:3.000  
##  Max.   :4.000   Max.   :1.0000   Max.   :1.00000   Max.   :5.000  
##                                                                    
##      v_prev      
##  Min.   :0.1530  
##  1st Qu.:0.5278  
##  Median :0.5481  
##  Mean   :0.5550  
##  3rd Qu.:0.5830  
##  Max.   :0.6927  
##                  
```

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```
polls_subset$v_prev <- polls_subset$v_prev*100 #rescale
polls_subset$region_label <- factor(polls_subset$region,levels=1:5,
                                    labels=c("NE","S","N","W","DC"))
#we consider DC as a separate region due to its distinctive voting patterns
polls_subset$edu_label <- factor(polls_subset$edu,levels=1:4,
                                 labels=c("No HS","HS","Some College","College Grad"))
polls_subset$age_label <- factor(polls_subset$age,levels=1:4,
                                 labels=c("18-29","30-44","45-64","65+"))
#the data includes states but without the names, which we will need,
#so let's grab that from R datasets
data(state)
#"state" is an R data file (type ?state from the R command window for info)
state.abb #does not include DC, so we will create ours
```

```
##  [1] "AL" "AK" "AZ" "AR" "CA" "CO" "CT" "DE" "FL" "GA" "HI" "ID" "IL" "IN" "IA"
## [16] "KS" "KY" "LA" "ME" "MD" "MA" "MI" "MN" "MS" "MO" "MT" "NE" "NV" "NH" "NJ"
## [31] "NM" "NY" "NC" "ND" "OH" "OK" "OR" "PA" "RI" "SC" "SD" "TN" "TX" "UT" "VT"
## [46] "VA" "WA" "WV" "WI" "WY"
```

```
#In the polls data, DC is the 9th "state" in alphabetical order
state_abbr <- c (state.abb[1:8], "DC", state.abb[9:50])
polls_subset$state_label <- factor(polls_subset$state,levels=1:51,labels=state_abbr)
rm(list = ls(pattern = "state")) #remove unnecessary values in the environment
```

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```
###### View properties of the data
head(polls_subset)
```

```
##      org survey bush state edu age female black region   v_prev region_label
## 1 cbsnyt   9158   NA     7   3   1      1     0      1 56.66333           NE
## 2 cbsnyt   9158    1    39   4   2      1     0      1 52.65667           NE
## 3 cbsnyt   9158    0    31   2   4      1     0      1 56.41667           NE
## 4 cbsnyt   9158    0     7   3   1      1     0      1 56.66333           NE
## 5 cbsnyt   9158    1    33   2   2      1     0      1 52.43666           NE
## 6 cbsnyt   9158    1    33   4   4      1     0      1 52.43666           NE
##      edu_label age_label state_label
## 1 Some College     18-29          CT
## 2 College Grad     30-44          PA
## 3           HS       65+          NJ
## 4 Some College     18-29          CT
## 5           HS     30-44          NY
## 6 College Grad       65+          NY
```

```
dim(polls_subset)
```

```
## [1] 2193   14
```

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```r
###### View properties of the data
str(polls_subset)
```

```
## 'data.frame':    2193 obs. of  14 variables:
##  $ org         : Factor w/ 1 level "cbsnyt": 1 1 1 1 1 1 1 1 1 1 ...
##  $ survey      : int  9158 9158 9158 9158 9158 9158 9158 9158 9158 9158 ...
##  $ bush        : int  NA 1 0 0 1 1 1 1 0 0 ...
##  $ state       : int  7 39 31 7 33 33 39 20 33 40 ...
##  $ edu         : int  3 4 2 3 2 4 2 2 4 1 ...
##  $ age         : int  1 2 4 1 2 4 2 4 3 3 ...
##  $ female      : int  1 1 1 1 1 1 0 1 0 0 ...
##  $ black       : int  0 0 0 0 0 0 0 0 0 0 ...
##  $ region      : int  1 1 1 1 1 1 1 1 1 1 ...
##  $ v_prev      : num  56.7 52.7 56.4 56.7 52.4 ...
##  $ region_label: Factor w/ 5 levels "NE","S","N","W",..: 1 1 1 1 1 1 1 1 1 1 ...
##  $ edu_label   : Factor w/ 4 levels "No HS","HS","Some College",..: 3 4 2 3 2 4 2 2 4 1 ...
##  $ age_label   : Factor w/ 4 levels "18-29","30-44",..: 1 2 4 1 2 4 2 4 3 3 ...
##  $ state_label : Factor w/ 51 levels "AL","AK","AZ",..: 7 39 31 7 33 33 39 20 33 40 ...
```

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What's next?

Move on to the readings for the next module!