IDS 702: Module 2.1

Odds, odds ratios, and relative risks

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INTRODUCTION

- So far, our response variables have been continuous.
- Sometimes, we would also like to build models for binary outcome variables. For example,
  - $Y = 1$: healthy, $Y = 0$: not healthy
  - $Y = 1$: employed, $Y = 0$: not employed
  - $Y = 1$: win, $Y = 0$: lose
- Often, we want to predict or explain the binary outcome variable from several predictors.
- Linear regression is NOT appropriate, because normality for the response variable (and errors) makes no sense in this case.
- This brings us to logistic regression, the most popular model for binary outcomes.
- First let's review relative risk, odds and odds ratios.

Absolute risk and relative risk

- Y: binary response variable, X: binary predictor

	$Y = 1$	$Y = 0$
$X = 1$	a	b
$X = 0$	c	d

- Absolute risk of $Y = 1$ for level $X = 1$: $\frac{a}{(a+b)}$
- Absolute risk of $Y = 1$ for level $X = 0$: $\frac{c}{(c+d)}$
- Relative risk (RR): $\frac{a/(a+b)}{c/(c+d)}$
- Relative risk is a ratio of two probabilities.

Give an example of an application where you think relative risk might be useful.

Odds and odds ratio

- Y: binary response variable, X: binary predictor

	$Y = 1$	$Y = 0$
$X = 1$	a	b
$X = 0$	c	d

- Odds of $Y = 1$ for level $X = 1$: $\frac{a}{b}$
- Odds of $Y = 1$ for level $X = 0$: $\frac{c}{d}$
- Odds ratio (OR): $\frac{a/b}{c/d}$
- Odds ratio is a ratio of two odds.

Give an example of an application where you think odds or odds ratio might be useful.

Probabilities and odds: motivating example

- Physicians' Health Study (1989): randomized experiment with 22071 male physicians at least 40 years old.
- Half the subjects were assigned to take aspirin every other day.
- The other half were assigned to take a placebo pill.
- Broad goal: determine whether aspirin decreases cardiovascular mortality.
- Here are the number of people in each cell of the contingency table:

	Heart attack	No heart attack
Aspirin	104	10933
Placebo	189	10845

Absolute risk and relative risk for physicians health study

- Physicians Health Study

	Heart attack	No heart attack
Aspirin	104	10933
Placebo	189	10845

- Relative risk of a heart attack when taking aspirin versus when taking a placebo equals

$$\text{RR} = \frac{104/(104+10933)}{189/(189+10845)} = 0.55$$

- Odds of having a heart attack when taking aspirin over odds of a heart attach when talking a placebo (odds ratio)

$$\text{OR} = \frac{104/10933}{189/10845} = 0.546$$

Interpreting odds ratios and relative risks

	$Y = 1$	$Y = 0$
$X = 1$	a	b
$X = 0$	c	d

- When the variables X and Y are independent

$$OR = 1; \qquad RR = 1$$

- When subjects with level $X = 1$ are more likely to have $Y = 1$ than subjects with level $X = 0$, then

$$OR > 1; \qquad RR > 1$$

- When subjects with level $X = 1$ are less likely to have $Y = 1$ than subjects with level $X = 0$, then

$$OR < 1; \qquad RR < 1$$

Relative risk vs. absolute risk: smoking and lung cancer

- Small or large values of relative risk may or may not be significant depending on the base rate.
- Thus, it can be more helpful or meaningful to present both the absolute risk and RR.
- For example,
  - Percentage of smokers who get lung cancer: 8% (conservative estimate)
  - Relative risk of lung cancer for smokers: 800%
  - That is, getting lung cancer is not commonplace, even for smokers but, smokers'chances of getting lung cancer are much, much higher than non-smokers' chances.
  - The absolute risk helps place the RR in context.

What's next?

Move on to the readings for the next module!